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2007

STATISTICS (Optional) 000074

सांख्यिकी (वैकल्पिक)

Time : 3 hours Maximum Marks : 200

Note : (i) *In all attempt* ***Five*** *Questions.*

(ii) *Question No. 1 is* ***Compulsory.***

(iii) *Of the remaining Questions, Attempt* ***Any four*** *by selecting* ***One Question*** *from each section.*

(iv) *Numbers of optional questions upto the prescribed number in the order in which questions have been solved, will only be assessed and excess answers of the question/s will not be assessed.*

(v) *Statistical and logarithmic tables will be supplied on request.*

(vi) *Use of your own simple electronic calculator is allowed.*

(vii) *Candidate should not write roll number, any names (including their own), signature, address or any indication of their identity anywhere inside the answer book otherwise he will be penalised.*

1. Attempt any **FOUR** of the following :

(a) Let X and Y be two independent gamma variates with parameters (α,λ_1) and (α,λ_2) respectively. Show that $(X+Y)$ is a gamma variate with parameters $(\alpha,\lambda_1+\lambda_2)$. Is $(X-Y)$ a gamma variate ? Justify your answer. **10**

(b) State the linear model used in the analysis of Latin Square Design (LSD). Partion the total sum of squares into various components in case of LSD. **10**

(c) Explain the method of maximum likelihood for estimation of a parameter. In random sampling from Normal population N (μ, σ^2), find the maximum likelihood estimator for μ when σ^2 is known. **10**

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(d) Solve following Linear Programming Problem (LPP) graphically

Minimise $Z = 2x_1 + x_2$

Subject to $5x_1 + 10x_2 \leq 50$

$x_1 + x_2 \geq 1$

$x_2 \leq 4$

$x_1, x_2 \geq 0$ **10**

(e) Define X^2- variate with n degrees of freedom.

Let Xi (i = 1, 2, ..., 9) be independent normal variates with E(Xi) = 0 and V(Xi) = i^2

Find :

$$P\left[\left(X_1^2 + \frac{X_2^2}{2^2} + \frac{X_3^2}{3^2} + \cdots + \frac{X_9^2}{9^2}\right) \geq 5.38\right]$$ **10**

SECTION - A

2. Answer the following subquestions :

(a) State the axioms of probability. With usual notations, prove that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Deduce a similar result for P (A ∪ B ∪ C) where C is one more event. **15**

(b) For the normal distribution, find

(i) Quartile deviation

(ii) Mean deviation about mean

Hence show that the quartile deviation the mean deviation and standard deviation are approximately 10 : 12 : 15. **15**

(c) Obtain the moment generating function (mgf) and cumulant generating function (cgf) of the exponential distribution with mean θ. Hence obtain mean and variance of the distribution. **10**

3. **Answer the following subquestions :**

(a) Define distribution function of a two dimensional continuous random variable (X, Y). State its important properties. **15**

(b) (i) A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as Poisson distribution with mean 1.5. Calculate the proportion of days on which (1) neither car is used and (2) the proportion of days on which some demand is refused. **8**

(ii) Out of 100 people in a certain village 40 always tell the truth and remaining always lie. A sample of 10 persons is chosen from these people. What is the probability that the sample will contain (1) three liers (2) no liers ? **7**

(c) Define Student's t - variate. Derive the probability density function (pdf) of Student's - t - distribution with n degrees of freedom. **10**

SECTION - B

4. **Answer the following subquestions :**

(a) Describe simple random sampling method. In simple random sampling without replacement (SRSWOR), show that sample mean is an unbiased estimator of population mean. Also obtain as expression for the variance of the sample mean. **15**

(b) What is meant by (i) main effect and (ii) interaction effects in a 2^3- factorial experiment ? Derive the expression for main effect of a factor. **10**

(c) In a trivariate distribution :

$\sigma_1 = 2,\ \sigma_2 = \sigma_3 = 3,\ r_{12} = 0.7,\ r_{23} = r_{31} = 0.5$

Find : (i) $r_{23.1}$ (ii) $R_{1.23}$ (iii) $\sigma_{1.23}$ **15**

5. **Answer the following subquestions :**

(a) Describe the systematic sampling method. Obtain an estimator of the population mean and variance of this estimator under systematic sampling. **15**

(b) Define efficiency of a design. Obtain an expression for the efficiency of LSD relative to CRD. **10**

(b) What is an Index Number ? Discuss the problems which arise while (i) selecting a base period (ii) selecting commodities in the construction of price index number. 15

(c) Explain in detail the $\overline{X}$ and R charts. What purposes do they serve ? State the criteria for detecting lack of control in $\overline{X}$ and R charts. 15

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(c) Let X and Y have bivariate normal distribution with parameters $\mu_x = 5$, $\mu_y = 10$,

$\sigma_x^2 = 1$, $\sigma_y^2 = 25$

and corr (X,Y) = ϱ

(i) Find the value of $\varrho (> 0)$ when $P(4<Y<16/X=5) = 0.954$

(ii) If $\varrho = 0$, find $P[X+Y \leq 16]$ **15**

SECTION - C

6. Answer the following subquestions :

(a) State and prove Cramer-Rao inequality. **15**

(b) On the basis of random sample of size n from N (μ, σ^2) obtain confidence internal for σ^2. When (i) μ is known (ii) μ is not known. **10**

(c) (i) Let P be the probability that a coin will fall head in a single toss. In order to test $H_0 = P = \frac{1}{2}$ against $H_1 = P = \frac{3}{4}$ the coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of Type I error and power of the test. **8**

(ii) A sample analysis of examination results of 200 MBA's was made. It was found that 46 students had failed 68 secured a third division, 62 secured a second division and the rest were placed in first division. Are these figures commensurate with the general examination result which is in the ratio 4 : 3 : 2 : 1 for various categories respectively ? **7**

7. Answer the following subquestions :

(a) Let X_1, X_2 and X_3 be a random sample of size 3 from a population with mean μ and variance σ^2. Let T_1, T_2 and T_3 are the estimators used to estimate the mean μ, where

$$T_1 = X_1 + X_2 - X_3 \qquad T_2 = 2X_1 + 3X_3 - 4X_2 \qquad T_3 = \frac{1}{3}(\lambda X_1 + X_2 + X_3)$$

(i) Are T_1 and T_2 unbiased estimators ?

(ii) Find the value of λ such that T_3 is an unbiased estimator for μ

(iii) Which is the best estimator ? **15**

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(b) What is interval estimation of a parameter ? Give the 95% confidence interval for the mean of the normal distribution, when its variance is known. 10

(c) (i) Define the terms :

(1) Simple hypothesis (2) Composite hypothesis

(3) Critical region (4) Level of significance 8

(ii) Write a short note on Wilcoxon signed rank test for paired observations. 7

SECTION - D

8. Answer the following subquestions :

(a) Define the following terms with reference to network analysis.

(i) Earliest start time (ii) Earliest finish time

(iii) Total float (iv) Critical path

(v) Slack 10

(b) What is Time Series ? Describe chief components of time series. 15

(c) Let double sampling plan be $N=5000$, $n_1=50$, $n_2=100$, $C_1=1$, $C_2=3$

Obtain (i) O.C. Curve (ii) AOQ Curve 15

9. Answer the following subquestions :

(a) Obtain an initial basic feasible solution to the following transportation problem using North-West corner method.

Origin	Destination			Availability
	A	B	C	
I	50	30	220	10
II	90	45	170	30
III	250	200	50	40
Demand	40	20	20	-

Is this solution optimal ? 10

P.T.O.