ATm

2007

Mathematics (Optional)

000614

गणित (वैकल्पिक)

Time : 3 hours **Maximum Marks : 200**

Note :
(i) *In all attempt Five Questions.*

(ii) *Question No. 1 is Compulsory.*

(iii) *Of the remaining Questions. Attempt **Any four** by selecting **one Question** from each section.*

(iv) *Numbers of optional questions upto the prescribed number in the order in which questions have been solved, will only be assessed and excess answers of the questions/s will not be assessed.*

(v) *Candidate should not write roll number, any names (including their own), signature, address or any indication of their identity anywhere inside the answer book otherwise he will be penalised.*

1. ***Answer any four of the following :***

(a) Complete one iteration of Simplex method, given the problem : **10**

minimize $2x_1 + 4x_2 + x_3$

subject to $x_1 + 2x_2 - x_3 \leq 5$

$2x_1 - x_2 + 2x_3 = 2$

$-x_1 + 2x_2 + 2x_3 \geq 1$

$x_1, x_2, x_3 \geq 0$

Is the solution at the end of first iteration optimal ? Give reasons.

P.T.O.

(b) If $f(n)$, $g(n)$, $n \in \mathbb{N}$ are such that - 10

$$f(n) > 0,\ g(n) > 0,\ \underset{n\to\infty}{Lt}\, g(n) = \infty,\ \underset{n\to\infty}{Lt}\, f(n) = \infty \text{ and } \underset{n\to\infty}{Lt} \frac{g(n)+3}{f(n)} = \frac{1}{32}.$$

Prove that the series -

$$\sum_{1}^{\infty} \left(\frac{g(n)}{f(n)}\right)^{\frac{2n}{3}}$$

is convergent.

(c) Derive the equations : 10

$$F_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

and $F_z = m\ddot{Z}$

for a body moving under influence of force $\bar{F}$.

Here (r, θ, z) are the cylindrical co-ordinates.

(d) Consider the Algorithm : 10

ALGORITHM XYZ (m, n)

1. r ← m mod n
2. if r = 0 GOTO 6
3. m ← n
4. n ← r
5. GOTO STEP 1 .
6. return n

for inputs m, n as positive integers and m > n.

Provide a DRY RUN for m = 51, n = 33. What does the function actually compute? Provide adequate justification for your answer.

(e) Show that the following alternating series is conditionally convergent. 10

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots\ldots$$

Section - A

2. (a) Define a Group. When do you say it is abelian ? **20**

If G is a group, prove that $(a.b)^2 = a^2.b^2$ for $\forall$ a, b $\in$ G iff G is abelian.

(b) If $V = R^3 = \{ (x, y, z) : x, y, z \in R \}$ and **20**

W = subset of V such that $x - 2y - 3z = 0$,

then show that

(i) V is a 3-dim vector space with R as scalars.

and (ii) W is a subspace of V with dim (W) = 2.

Further, if the equation defining W, i.e., $x - 2y - 3z = 0$ is changed to $x - 2y - 3z = 5$, will it still remain a subspace.

3. (a) (i) Define a Ring, Ideal and a Eucledian domain. **20**

(ii) Prove that a non empty subset H of a group G is a subset iff

$$a \in H, b \in H \Rightarrow a_0\, b^{-1} \in H .$$

(b) If S = $\{ v_1 , v_2 , \ldots , v_n \}$ is a set of n-linearly independent vectors belonging to vector space V(F) and dim V = n then show that - **20**

(i) any u $\in$ V can be expressed as a linear combination of elements of S and

(ii) such a representation is unique.

Section - B

4. (a) (i) Evaluate the limits - **20**

$$\underset{x \to 0}{Lt} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}} \text{ and}$$

$$\underset{x \to 0}{Lt}\; x^{n-3} \ln x^{x^3} \quad (n>3, \text{ integer})$$

(ii) State and prove Cauchy's MVT.

(b) Test the convergence of the integrals - **20**

(i) $\int_a^\infty \frac{dx}{x^n}$ a > 0, n $\in$ R and

(ii) $\int_a^\infty \frac{dx}{n^x}$ a>0, n>0

(b) (i) Evaluate using $h = \frac{1}{6}$ and simpson's $\frac{3}{8}$ th Rule $\int_0^1 \frac{1}{1+x}\,dx$. 20

(ii) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.1$ if

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	0	1.280	0.554	1.296	2.432	4.0

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5. (a) (i) Show that $(|x|<1)$ **20**

$$\sqrt{1+x} = f(x) + \frac{x^3}{16(1+\theta x)^{5/2}}$$

with $0 < \theta < 1$ and $f(x)$ a quadratic expression in x.

(ii) Find maximum of

$v = xyz$ $x>0,\ y>0,\ z>0.$

provided that $xy + yz + yx - a = 0.$

(b) If P, Q are partitions of [a, b], then show that : **20**

(i) L (f, Q) ≤ L (f, P) and

(ii) U (f, Q) ≥ U (f, P)

whenever P is finer than Q, f is a Reimann integrable function over [a, b] and L, U are the lower and upper Darboux sums.

Section - C

6. (a) Show that ingeneral two tangents can be drawn to a circle from a point lying on the exterior of the circle, and lying in the plane of the circle. **20**

Derive the equation of the Director Circle.

(b) Show that area bounded by a simple closed curve C is given by - **20**

$$\frac{1}{2}\oint_C (x\,dy - y\,dx)$$

Demonstrate its applicability by calculating the area bound by the curve :

$x = a\cos\theta,\ y = b\sin\theta,\ 0 \le \theta \le 2\pi$

7. (a) If $P(t_1)$ and $Q(t_2)$ are the end points of a focal chord of the standard parabola $y^2 = 4ax$, show that $t_1 t_2 = 1$. Here t is the parameter in the parametric representation. **20**

$x = at^2$ and $y = 2at$ of the conic.

Further show that slopes of OP and OQ is constant where O is the origin.

(b) Consider the following double integral

$$\int_0^1 \int_y^{y+1} x^2 y \, dx dy = \iint_R x^2 y \, dy$$ 20

(i) Describe the region R

(ii) Change the order of integration

(iii) Evaluate the integral

Section - D

8. (a) Find the complete solution of - 20

$$y^2 \left(1 + \left(\frac{dy}{dx} \right)^2 \right) = R^2$$

Show that solution is a family of circles.

Equation has singular solutions as wel. Find them.

(b) (i) Find the value of x for which $\log_e x = \cos x$. 20

correct upto 5 decimal places.

(ii) Population of a town in a census is recorded as :

Year	1950	1960	1970	1980	1990
Population in '000'	46	66	81	93	101

Find the population (estimate) for the year 1985 by interpolation.

9. (a) (i) Given F(D) y=($aD^2 + 2bD + c$) y with a, b, c constants and $D \equiv d/dx$ establish following identities. 20

1. $F(D)\, e^{kx} = e^{kx}\, F(K)$
2. $F(D^2) \sin ax = \sin ax \; F(-a^2)$
3. $F(D)\, e^{kx}\, u(x) = e^{kx}\, F(D+K)\, u(x)$

(ii) Solve $\frac{dy}{dx} + xy = x^3 y^3$