Syllabus for the post of
Assistant Professor- Mathematics, Maharashtra Education Services, Group - A (Collegiate Branch)

Steps of Exam: Written Exam - 200 Marks Interview - 50 Marks

Level: - Degree No. of Questions: - 100

Medium: English No. of Marks: - 200

Nature of Paper - Objective Type Duration: - 1 hour

Final merit list will be prepared by considering the marks obtained in Written test & Interview.

SYLLABUS

1 **Real Analysis :** Riemann integrable functions; improper integrals, their convergence and uniform convergence. Eulidean space R", Bolzano-Weierstrass theorem, compact Subsets of R", Heine-Borel T\theorem, Fourier series.

Continuity of functions on R", Differentiability of F:R"-R^m. Properties of differential, partial and directional derivatives, continuously differentiable functions. Taylor's series.

Inverse function theorem, Implieit function theorem.
Integral functions, line and surface integrals, Green's theorem, Stoke's theorem.

2 **Complex Analysis :** Cauchy's theorem for convex regions. Power series representation of Analtic functions. Liouville's theorem, Fundamental theorem of algebra Riemann's theorem on removable singularities, maximum modulus principle. Schwarz Iemma, Open Mapping theorem, Casorattl-Weierstrass-theorem, Weierstrass's theorem on uniform convergence on compact sets, Bilinear transformations, Multivalued Analytic Functions, Riemann Surfaces.

3 **Algebra :** Symmetric groups, Alternating groups, Simple groups, Rings, Maximal Ideals, Prime Ideals, Integral domains Euclidean domains, principal Ideal domains, Unique Factorisation domains, quotient fields, Finite fields, Algebra of Linear Transformations, Reduction of matrices to Canonical Forms, Inner Product Spaces, Orthogonality, Quadratic Forms, Reduction of quadratic forms.

4 **Advanced Analysis :** Elements of Metric Spaces, Convergence, continuity, compactness, Connectedness, Weierstrass's approximation Theorem, Completeness, Bare category theorem, Labesgue measure, Labesgue Integral, Differentiation and Integration.

5 **Advanced Algebra :** Conjugate elements and class equations of finite groups, Sylow theorems, solvable groups, Jordan Holder Theorem, Direct Products, Structure Theorem for finite abelian groups, Chain conditions on Rings; Characteristic of Field, Field extensions, Elements of Galois theory, solvability by Raducals, Ruler and compass construction.

6 **Functional Analysis :** Banach Spaces Hahn-Banach Theorem, Open mapping and closed Graph Theorems. Principal of Uniform boundedness, Boundedness and continuity of Linear Transformations, Dual Space, Embedding in the second dual, Hilbert Spaces, Projections.
Orthonormal Basis, Riesz - representation theorem, Bessel's Inequality, parsaval's identity, self adjoined operators, Normal Operators.

7 **Topology :** Elements of Topological Spaces, Continuity, convergence, Homeomorphism, Compactness, Connectedness, Separation Axioms, First and Second Countability, Separability, Subspaces, Product Spaces, quotient spaces. Tychonoft's Theorem, Urysohn's Metrization theorem, Homotopy and Fundamental Group.

8 **Discrete Mathematics :** Partially ordered sets, Lattices, Cornplete Lattices, Distrbutive lattices, Complements, Boolean Algebra, Boolean Expressions, Application to switching circuits, Elements of Graph Theory, Eulerian and Hamiltonian graphs, planar Graphs, Directed Graphs, Trees, Permutations and Combinations, Pigeonhole principle, principle of Inclusion and Exclusion, Derangements.

9 **Ordinary and partial Differential Equations :** Existence and Uniqueness of solution dy/dx = f (x,y) Green's function, sturm Liouville Boundary Value Problems, Cauchy Problems and Characteristics, Classification of Second Order PDE, Separation of Variables for heat equation, wave equation and Laplace equation, Special functions.

10 **Number Theory :** Divisibility; Linear diophantine equations. Congruences. Quadratic residues; Sums of two squares, Arithmatic functions Mu, Tau, and Signa (and).

11 **Machanics :** Generalise coordinates; Lagranges equation; Hamilton's cononical equations; Variational Principles-Hamilton's principles and principles of least action; Two dimensional motion of rigid bodies; Euler's dynamical equations for the motion of rigid body; Motion of a rigid body about an axis; Motion about revolving axes.

12 **Elasticity :** Analysis of strain and stress, strain and stress tensors; Geometrical representation; Compatibility conditions; Strain energy function; Constritutive relations; Elastic solids "Hookes law; Saint-Venant's principle, Equations of equilibrium; Plane problem-Airy's stress function vibrations of elastic, cylindrical and spherical media.

13 **Fluid Mechanics :** Equation of continuity in fluid motion; Euler's equations of motion for perfect fluids; Two dimensional motion complex potential; Motion of sphere in perfect liquid and montion of liquid past a sphere; vorticity; Navier - Stokes's equations for viscous flows - some exact solutions.

14 **Differetial Geometry :** Space curves-their curvature and torsion; Serret Frehat Formula; Fundamental theorem of space curves; Curves on surfaces; First and second fundamental form; Gaussian curvatures; Principal directions and principal curvatures; Goedesics, Fundamental equations of surface theory.

15 **Calculus of Variations :** Linear functionals, minimal functional theorem, general variation of a functional, Euler-Lagrange equation; Variational methods of boundary value problems in ordinary and partial differential equations.

16 **Linear Integral Equations :** Linear Integral Equations of the first and second kind of Fredholm and Volterra type; solution by successive substitutions and successive approximations; Solution of equations with separable kernels; The Fredholm Alternative; Holbert-Schmidt theory for symmetric kernels.

17 **Numerical analysis :** Finite differences, Interpolation; Numerical solution of algebric equation; Iteration; Newton-Raphason Method; Solution on Linear system; Direct method; Gauss elminaiton method; Matrix-Inversion eigenvalue problems; Numerical differentiation and integration.

Numerical solution of ordinary differential equation; iteration method, Picard's method Euler's method and improved Euler's method.

18 **Integral Transform :** Laplace transform; Transform of elementary functions, Transform of Derivatives, Inverse Transform, Convolution Theorem, Applications, Ordinary and Partial differential equations; Fourier transform; sine and cosine transform, Inverse Fourier Transform, Application to ordinary and partial differential equations.

19 **Mathematical Programming :** Revised simplex method, Dual simplex method, Sensitivity analysis and parametric linear programming. Kuhn-Tucker conditions of optimality. Quadratic programming; methods due to Beale, Wofle and Vandepanne, Duality in quadratic programmming, self duality, Integer programming.

20 **Measure Theory :** Measurable and measure spaces : Extension of measures,signed measures, Jordan-Hahn decomposition theorems. Integratiuon, monotone convergence theorem, Fatou's lemma, dominaated convergence theorem. Absolute continuity, Radon Nikodym theorem, Product measures, Fubini's theorem.

21 **Probability :** Sequences of events and random variables: Zero-one laws of Borel and Kolmogorov.

Almost sure convergence, convergence in mean square, Khintchine's weak law of large numbers; Kolmogorov's inequality, strong law of large numbers.

Convergence of series of random variables, three-series criterion. Central limit theorems of Liapounov and Lindeberg-Feller. Conditional expectation, martingales.

22 **Distribution Theory :** Properties of distribution functions and characteristic functions; continuty theorem, inversion formula, Representation of distribution function as a mixture of discrete and continuous distribution functions; Convolutions, marginal and conditional distributions of bivariate discrete and continuous distributions.

Relations between characteristic functions and moments; Moment inequalities of Holder and Minkowski.

23 **Statistical Inference and Decision Theory :** Statistical decision problem : non-randomized, mixed and randomized decision rules; risk function admissibility, Bayes rules, minimax rules, least favourable distributions, complete class and minimal complete class. Decision problem for finite parameter space. Convex loss function. Role of sufficiency.

Admissible, Bayes and minimax estimators; illustrations. Unbiasedness. UMVU estimators.

Families of distributions with monotone likelihood property, exponential family of distributions. Test of a simple hypothesis against a simple alternative from decision-theoretic viewpoint. Tests with Neyman structure. Uniformly most powerful unbiased tests. Locally most powerful tests.

Inference on location and scale parameters; estimation and tests. Equivariant estimators. Invariance in hypothesis testing.

24 **Large sample statistical methods :** Various modes of convergence. Op and op, CLT, Sheffe's theorem, Polya's theorem and Slutsky's theorem. Transformation and variance stabilizing formula. Asymptotic disribution of function of sample moments. Sample quantiles. Order statistics and their functions. Tests on correlations, coefficient of variation, skewness and kurtosis. Pearson Chi-square, contingency Chi-square and likelihood ratio statistics. U-statistics consistency of Tests. Asymptotic relative efficiency.

25 **Multivariate Statistical Analysis :** Singular and non-singular multivariate distributions. Characteristics functions. Multivariate normal distributions, margrinal and conditional distributions; distribution of linear forms, and quadratic forms, Cochran's theorem. Inference on parameters of multivariate normal distributions, one-population and two population cases. wishart distribution. Hotellings T2, Mahalanobis D2 Discrimination analysis, Principal components, Canonical correlations, Cluster analysis

26 **Linear Models and Regression :** Standard Gauss-Markov models; Estimability of parameters; best linear unbiased extimates(BLUE); Method of least squares and Gauss-Markovtheorem; Variance-covariance matrix of BLUES.

Tests of linear hypothesis; One-way and two-way classifications. Fixed, random and mixed effects models (two-way classifications only); variance components, Bivariate and multiple linear regression; Polynomial regression; use of ortheogonal polynomials. Analysis of covarance. Linear and nonlinear regression outliers.

27 **Sample Surveys :** Sampling with varying probability of selection, Hurwitz-Thompson estimator; PPS sampling: Double sampling. Cluser sampling. Non-sampling errors: Interpentrating samples. Multiphase sampling. Ratio and regression methods of estimation.

28 **Design of Experiments :** Factorial experiments, confounding and fractional replication.

Split and strip plot designs; Quasi-Latin square designs; Youden square. Design for study of response surfaces; first and second order designs.
Incomplete block designs; Balanced, connectedness and orthogonality, BIBD with recovery of inter-block information PBIBD with 2 associate classes. Analysis of series of experiments, estimation of residual effects. Construction of orthogonal-Latin squares, BIB designs, and confounded factorial designs.

Optimality criteria for experimental designs.

29 **Time-Series Analysis :** Discrete-parameter stochastic processes; strong and weak stationarity; autocovariance and autocorrelation. Moving average, autoregressive, autoregressive moving average and autoregressive intgegfrated moving average processes. Box-Jenkins models. Estimation of the parameters in ARIMA models; forecasting. Periodogram and correlogram analysis.

30 **Stochastic Processes :** Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution; branching processes; Random walk; Gambler's ruin.

Markov processes in continuous time; Poisson processes, birth and death processes, Wiener process.

31 **Demography and Vital Statistics :** Measures of fertility and mortality, period and Cohort measures.

Life tables and its applications; Methods of construction of abridged life tables. Application of stable population theory to estimate vital rates. Population projections. Stochastic models of fertility and reproduction.

32 **Industrial Statistics :** Control charts for variables and attributes; Acceptance sampling by attribites; single, double and sequential sampling plans; OC and ASN functions, AOQL and ATI; Acceptance sampling by varieties. Tolerance limits Reliability analysis: Hazard function, distribution with DFR and IFR; Series and parallel systems. Life testing experiments.

33 **Inventory and Queueing theory :** Inventory (S,s) policy periodic review models with stochastic demand. Dynamic inventory models. Probabilistic re-order point, lot size inventory system with and without lead time. Distribution free analysis. Solution of inventory problem with unknown density function. Warehousing problem. Queues: Imbedded markov chain method to obtain steady state solution of M/G/1, G/M/1 and M/D/C, Network models. Machine maintenance models. Design and control of queueing systems.

34 **Dynamic Programming and Marketing :** Nature of dynamic programming, Deterministic processes, Non-sequential discrete optimisation-allocation problems, assortment problems. Sequential discrete optimisation long-term planning problems, multi stage production processes. Functional approximations. Marketing systems, application of dynamic programming to marketing problems. Introduction of new product, objective in setting market price and its policies, purchasing under Fluctuating prices, Advertising and promotional decisions, Brands switching analysis, Distribution decisions.